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ORIGINAL ARTICLE

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Energy loss due to radiation in postmortem cooling

Part B: Energy balance with respect to radiation

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Abstract With the help of the law of Stefan and Boltzmann and a model for the cooling of exposed skin derived from the data of Lyle and Cleveland [7], the radiation energy loss E_R can be calculated according to the following formula:

$$E_R(t) = \varepsilon\, \sigma\, A_R \int_0^t ([(T_S(0) - T_E)\, e^{-Z' t} + T_E] - T_E{}^4)\, d\, t'$$

where ε represents the emissivity of the skin (0.98), σ the Stefan-Boltzmann constant, A_R the radiating surface area, $T_S(0)$ the skin temperature at death, T_E the environmental temperature and $Z' = 0.1017$ the gradient of the skin temperature curve.

Additionally, an energy loss due to conduction and convection E_C has to be taken into account. Comparing the energy losses due to radiation, conduction and convection with the decrease E_T of the thermal energy in the body, calculated from mean heat capacity (3.45 kJ/(kg °K)), body mass and decrease of mean body temperature, there is a surplus of energy in the very early postmortem period, which can be explained only by an internal source of energy E_I. Alltogether the following balance equation can be formulated:

$$E_T + E_I = E_R + E_C$$

Since the body temperature decreases in the early postmortem period, E_I can be estimated by: $E_I(t) \geq \max(E_R(t) - E_T(t), 0)$. The values obtained range up to 500 kJ for a medium sized (175 cm), medium weight (75 kg) body at an environmental temperature of 5 °C and are compatible with estimations of Lundquist [6] for supravital energy production by breakdown of glycogen.

Key words Postmortem cooling · Radiation · Conduction · Convection · Thermal energy · Supravital activity · Time since death

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Introduction

The contribution of thermal radiation to the energy loss in postmortem cooling is assessed differently in the forensic literature [3–5]. Part A of the paper [8] presented quantitative estimations of the energy loss due to radiation under standardized conditions using the law of Stefan and Boltzmann. The calculations showed that radiation considerably contributes to the cooling of a dead body.

In the very early postmortem period, the radiation energy loss exceeds the thermal energy loss, calculated from body mass, body heat capacity and the difference between body temperature at death and at time t during the cooling process. The amount of energy emitted by radiation cannot be explained by the loss of thermal energy alone. To maintain the energy balance according to the energy conservation law, a source of energy within the body has to be postulated.

In the present part B of the paper, this matter is quantitatively analysed, pointing out that in the early postmortem period an internal production of thermal energy, which may be explained by chemical processes in the supravital period, has to be assumed. A model for the balance of energies and powers is developed leading to formulae, by which it is possible to estimate a lower bound for the amount of the internal energy production post mortem.

Since the arguments presented are to some extent technical, the paper is structured into text and appendix. Strictly formal definitions and deductions are described in detail in the appendix and cited only by their number in the text.

Method

Power and energy model

The following model is developed for the description of the energetic circumstances in the postmortem cooling process:

The human body B in the present model can be completely described by the following quantities:

m: body mass in kg
c: specific heat capacity in kJ/(kg °K)
A_R: radiating surface area in m^2

The specific heat capacity is assumed as 3.45 kJ/(kg °K) [10]. The radiating surface area for a prone dead body is determined by reducing the Dubois surface area by 0.5 [2].

Let the death (cardiopulmonary arrest) of a human body B occur at time t = 0. The environmental temperature T_E at the time of death and for the time postmortem $t > 0$ is assumed to be constant (A1). Let the mean body temperature $T_M(t)$ of the body be defined, such that for all times $t \geq 0$ the thermal energy content $Q_T(t)$ of the body B is equal to the mean body temperature multiplied by body mass m and the specific heat capacity c (A2).

Let $T_C(t)$ be the core temperature and $T_S(t)$ the mean surface (or skin) temperature of the body at time t. The mean body temperature $T_M(t)$ can be estimated by determining a real number $0 \leq \gamma \leq 1$ taking into account the temperature gradient between the core of the body and its periphery. This is done by weighting the skin temperature T_S with $(1 - \gamma)$ and the core temperature T_C with γ (A3). According to physiological findings [2], the number γ can be assumed as 0.7 for the early postmortem period. Under the approximative presupposition, that all body types and body mass distributions can be modeled by linear contractions or inflations of one ‘prototype’, the number γ can be assumed to be independent of individual properties such as body mass and size.

Energies and powers in the model

All energies and corresponding powers are, by convention and for the sake of clarity, counted positive in all the following equations. This is possible, because the skin temperature T_S, falling to the environmental temperature T_E, is at all times assumed to be higher than T_E.

The following powers influence the process of postmortem cooling:

– Power $P_I(t)$ due to internal energy production.
  $P_I(t)$ denotes the production of thermal energy per unit of time in the body B by reactions due to supravital activity.

– Power $P_R(t)$ due to radiation.
  The loss of thermal energy by electromagnetic radiation is quantitatively anlysed in part A of the paper [8]. The power due to radiation $P_R(t)$ of the body B at time t can be determined by the law of Stefan and Boltzmann [1]. It is proportional to the difference of the fourth powers of T_S and T_E (A4). The constants of proportionality are the emissivity of the skin $\varepsilon = 0.98$ [2], the Stefan-Boltzmann constant $\sigma = 5.67 \times 10^{-8}$ W/(m^2 K^4) and the radiating surface area A_R.

– Power $P_C(t)$ due to conduction and convection.
  The direct transfer of thermal energy from the body B to the surrounding media by conduction and convection represents a further source of energy loss. The power $P_C(t)$ due to conduction and convection at time t can be described as a monotonously falling positive function of the difference between skin temperature T_S and environmental temperature T_E (A5).

Integrating over time the internal power $P_I(t)$, the radiation power $P_R(t)$ and the conductive/convective power $P_C(t)$ from the time of death (t = 0) to the time postmortem $t > 0$ leads to the corresponding energies $E_I(t)$, $E_R(t)$ and $E_C(t)$ (A6).

The model makes use of the energy conservation law in the following way:

The body B can be considered as a thermal reservoir with a thermal energy content Q_T. There are two kinds of energy transfer: metabolic processes as well as supravital activity supply thermal energy to the reservoir B via the power P_I; radiation, conduction and convection withdraw thermal energy from the reservoir B via the power $(P_R + P_C)$.

Before death ($t < 0$) and under normal circumstances (constant body temperature), the metabolic processes in the body supply as much energy per unit of time as is lost by radiation, conduction and convection per unit of time. After death ($t > 0$), since the energy intake by nutrition has ceased, the body B represents a one-sided open system.

In the early postmortem period, the mean body temperature T_M and correspondingly the skin temperature T_S and core temperature T_C continuously and monotonously decrease to the environmental temperature T_E.

The energy loss due to radiation E_R, conduction/convection E_C and the energy gain due to internal energy production E_I are balanced by the corresponding change of the content of thermal energy. In the early postmortem period, the thermal energy content Q_T decreases, except for special environmental or body conditions (e.g. rapid onset of decay) which are not subject of the estimations presented.

It is not possible to calculate the actual change of the thermal energy content due to radiative and conductive/convective heat loss in a time span $t_1 < t_2$ by simply inserting the difference of the mean body temperature $T_M(t_2) - T_M(t_1)$ in the definition for the thermal energy content Q_T (A2), because the thermal energy produced by supravital activity E_I during the time span $t_1 < t_2$ increases the mean body temperature $T_M(t_2)$. Thereby, the difference between the thermal energy content at the time of death and the time t postmortem is reduced. While the thermal energy content Q_T calculated from the decrease of the mean body temperarture T_M is equal to the energy content of the body, this would be valid for changes of the thermal energy content only if there was no internal energy production. We therefore define an *apparent* thermal energy change $E_T(t)$ at time t as the difference $Q_T(0) - Q_T(t)$ of the thermal energy content at time t = 0 and t (A7). Thus, it is possible to formulate a balance equation of the energies in the cooling process (A8). The energy ‘gain’ due to apparent thermal energy change E_T and internal energy E_I is balanced by the energy ‘loss’ due to radiation E_R and conduction/convection E_C. An analogous balance can be formulated for the corresponding powers (A9).

As already mentioned, all quantities in the power and energy balance equations have positive values. This convention can be applied since the energies due to radiation E_R and conduction/convection E_C strictly flow from the body to the environment.

Assumptions for the cooling process

The model contains the time-dependent temperatures $T_S(t)$, the skin temperature, and $T_M(t)$, the mean body temperature, as variables. To be able to derive quantitative statements, these two functions of time have to be expressed by calculable formulae.

As already presented in detail in Part A [8], the course of the skin temperature $T_S(t)$ for the time postmortem can be described by a simple single-exponential model (A10). The difference between skin temperature $T_S(t)$ and the environmental temperature T_E is proportional to a falling exponential function of time with the difference between skin temperature at time of death and environmental temperature as factor of proportionality. The gradient Z' was determined by a loglinear regression analysis of the temperature difference data of Lyle and Cleveland for exposed skin [7].

The mean body temperature $T_M(t)$ allows the calculation of the apparent thermal energy loss $E_T(t)$ of the body (A7) as a function of skin temperature T_S and and core temperature T_C (A11).

The starting temperature of the skin $T_S(0)$ is assumed as 33 °C [2], the starting temperature of the core of the body $T_C(0)$ as 37 °C. The time-dependent behaviour of the core temperature of the body

$T_C(t)$ is determined according to the double-exponential approach of Marshall and Hoare [9], Henßge [3] and Henßge and Madea [4], valid for the rectal temperature (A12). For estimative purposes it is sufficient to identify core and rectal temperature. The values for the coefficients Z and p are calculated, as advised by Henßge [3] for standard conditions, i.e. the naked body lying extended on the back on a thermally indifferent ground in still air in a closed room and without any sources of strong radiation:

$Z = (1.2815\ m^{-0,625} - 0.0284)\ h^{-1}$
$p = 5\ Z$

Where m denotes the pure number of the body weight in kg.

Lower bounds for the internal energy and power

For the time postmortem $t \geq 0$ the power balance equation (A9) of the model can be solved for $P_I(t)$ (A13). The power due to internal energy production P_I is at all times t equal to the power due to radiation and due to conduction/convection $P_R + P_C$ minus the thermal power P_T delivered by the pure cooling of the body. Since the amount of conductive and convective power P_C in the estimation presented can only assume positive values (the body is not warming up), the amount of internal power $P_I(t)$ *a fortiori* has to be greater than the amount of radiation power $P_R(t)$ minus the amount of thermal power $P_T(t)$ at time t (A14). Since this is valid for all times t, it can be transferred to the corresponding energies as integrals of the powers (A15).

The inequality ($E_I \geq E_R - E_T$) directly provides a lower bound LE_I for the internal energy (A16a, b). In analogy, a lower bound for the internal power LP_I (A17a, b) can directly be derived from the inequality of powers ($P_I \geq P_R - P_T$).

Improvement of the lower bounds for the internal power and internal energy

The lower bounds LP_I (A17a, b) and LE_I (A16a, b) of the internal power and internal energy can be improved by using the power due to conduction and convection P_C. On the one hand, the power due to radiation P_R falls rapidly with increasing time postmortem in a monotonous way (cf. Fig. 1). On the other hand, the thermal power P_T first rises starting from zero and then slowly falls towards zero again (cf. Fig. 1). Therefore, it is possible to determine a time t_{max} (A18), for which the surplus of the thermal power P_T over the power due to radiation P_R reaches a maximum. Since the balance equation for the powers (A9) is valid for all times t, it is also valid for t_{max}. Solving the power balance equation for the power due to conduction and convection P_C leads to an equation where P_C is balanced by the thermal power P_T minus radiation power P_R plus internal power P_I at time t_{max} (A19). Since the internal power P_I (originating from exothermal processes) cannot assume a negative value at time t_{max}, the amount of the power due to conduction/convection P_C at time t_{max} is greater than or equal to the difference $P_T(t_{max}) - P_R(t_{max})$ (A20). This inequality directly provides a lower bound for the power P_C at time t_{max}. Since the power due to conduction and convection P_C is assumed to be a monotonously falling function with time (A21), the lower bound is valid for the times before t_{max} as well (A22). By adding the amount of conductive/convective power P_C at time t_{max} to the lower bound LP_I (see in detail A23) it is now possible to derive an improved estimate of the lower bound for the internal power LPC_I (A24). As the internal energy E_I represents the time integral of the internal power P_I, the improved lower bound for the internal energy LEC_I can directly be deduced (A25).

Results

The following results should be understood as a rough estimation of the energy conditions based on the currently accessible experimental results (e.g. skin cooling, decrease of mean body temperature).

The curves given in the Figs. 1–3 are valid for a medium sized (175 cm) and standard weight (75 kg) body at an environmental temperature of 5 °C under standard conditions, i.e. the naked body lying extended on the back on a thermally indifferent ground in still air in a closed room without sources of strong radiation. Figure 1 presents the apparent thermal power P_T, the power due to radiation P_R, the lower bound for the internal power LP_I (as derived from the difference $P_R - P_T$ alone) and the improved lower

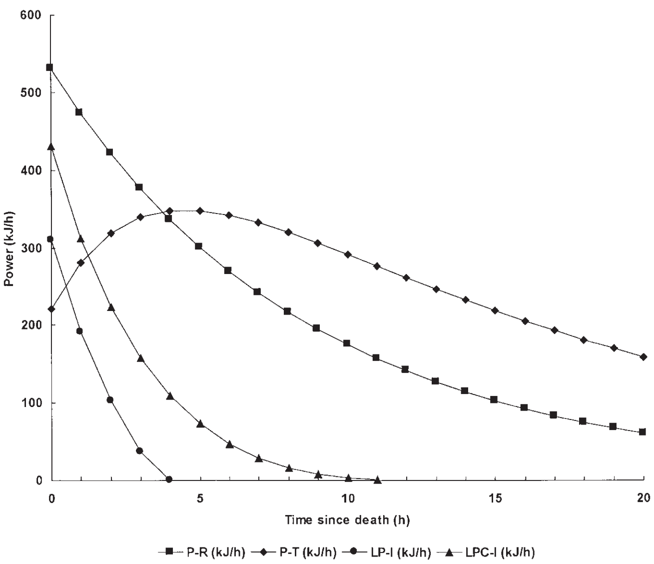

Fig. 1 Radiation power P_R, thermal power P_T and lower bound of internal power LP_I in kJ/h up to 20 h postmortem in a standard sized and standard weight body (175 cm, 75 kg) at an environmental temperature $T_E = 5$ °C

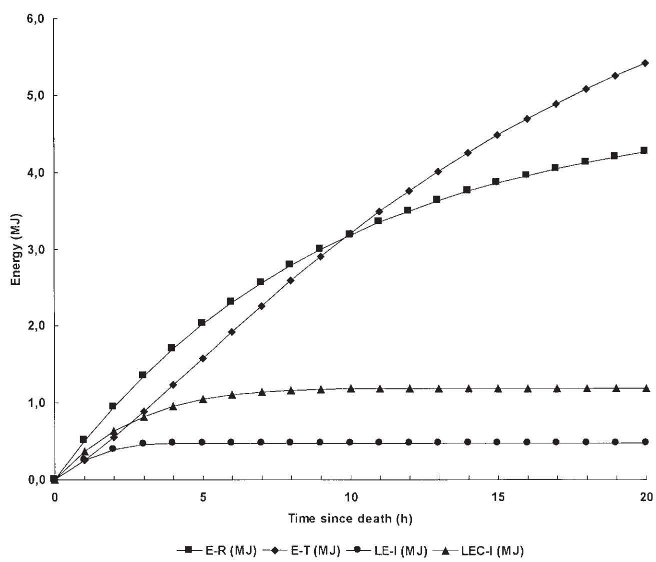

Fig. 2 Radiation energy E_R, thermal energy E_T and lower bound of internal energy LE_I in kJ up to 20 h postmortem in a standard sized and standard weight body (175 cm, 75 kg) at an environmental temperature $T_E = 5$ °C

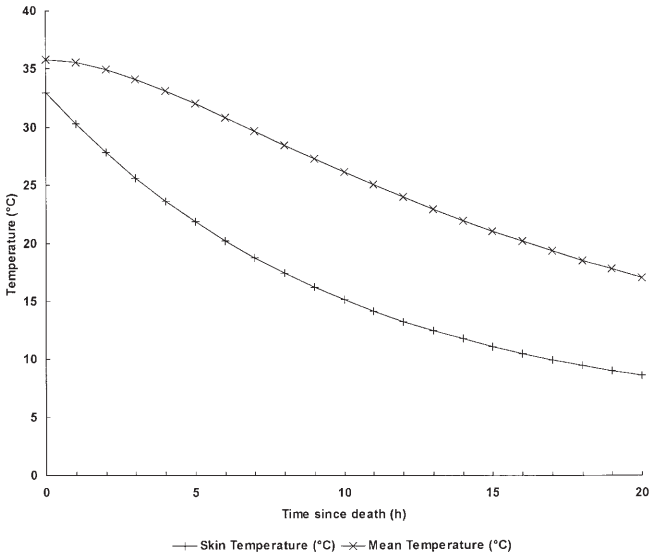

Fig. 3 Skin temperature and mean body temperature in °C up to 20 h postmortem for a standard sized and standard weight body (175 cm, 75 kg) at an environmental temperature T_E = 5 °C

bound LPC_I for the same time span. Figure 2 gives the time-dependent course of the corresponding energies E_T, E_R, LE_I and LEC_I. The time-dependent course of the skin temperature T_S and of the mean body temperature T_M during the first 20 h postmortem is shown in Fig. 3.

In Tables 1 and 2, the cumulative amounts of the energies E_R, E_T and LEC_I as well as the amounts of the corresponding powers are listed for non-standard body weight and body height in different environmental temperatures. The results are presented for small bodies (165 cm) of lean (50 kg), standard weight (65 kg) and overweight (80 kg) stature and for tall bodies (185 cm) of lean (70 kg), standard weight (85 kg) and overweight (100 kg) stature at environmental temperatures of 5 °C (Table 1) and 20 °C (Table 2) up to 10 h postmortem.

Discussion

In continuation of Part A of the paper [8], which gives an estimation of the amounts of radiation energy loss in postmortem cooling, part B compares the radiation energy loss to the apparent loss of the thermal energy calculated from body heat capacity, body mass and decrease of mean body temperature. As the amount of radiation emitted cannot be fully explained by the loss of apparent thermal energy, an energy, or power model was developed, including a source of internal energy production within the dead body.

The course of the skin temperature, necessary to calculate the radiation losses, is based on the values obtained by Lyle and Cleveland [7] from measurements at the forehead. Because of the greater thermal reservoir, the temperature of the skin on the trunk will most probably decrease more slowly. A slower decrease of the skin temperature leads to an increased radiation power (being dependent on the temperature difference between skin and environment to the fourth power) as well as to a higher heat loss due to conduction and convection. Consequently the calculations presented underestimate the lower bound for the internal energy production.

In the following, the dependence of the estimation of the internal energy E_I by means of the difference between the energies E_R and E_T on the selection of the weighting coefficient γ for determining the mean body temperature T_M (see A3) is discussed. According to its definition (see A16a) the lower bound $LE_I(t)$ for the internal energy E_I becomes zero at time t if for all times $t^* < t$ the apparent thermal energy $E_T(t^*)$ is greater than or equal to the radiation energy $E_R(t^*)$. Since the highest rates of internal energy production are reached in the very early postmortem period (close to t = 0), an analytical approach for determining a limit coefficient γ_{lim} is possible (see in detail B1–7). Inserting the values for the standard case presented in Figs. 1–3 (body weight: 75 kg, body height: 175 cm, T_E = 5 °C) leads to:

$\gamma_{lim} = 0.393$

It can therefore be concluded that the weighting ratio between core temperature T_C and skin temperature T_S assumed in the presented estimations according to physiological standard conditions [2] with a weighting coefficient $\gamma = 0.7$ has to be almost reversed to make the presented method of estimating the internal energy production impossible.

The temperature weighting coefficient γ for the determination of the mean body temperature was further assumed to be constant with time. This assumption is supported by the following consideration: the human body is approximated as consisting of two homogeneous components, a peripheral and a central one with the specific heat capacities c_p for the periphery and c_C for the centre. Let the temperature of the periphery at time t be $T_S(t)$, the temperature of the centre $T_C(t)$. The thermal energy content of the whole body at the time t can now be calculated in two ways. Firstly, the thermal energy content can be calculated by multiplying mean body temperature T_M with overall body mass and overall specific heat capacity (B8). Secondly, the energy content can be replaced by the sum of the energy contents Q_P of the peripheral and Q_C of the central component (B9). Q_P is proportional to T_S with the factors mass m_P and specific heat capacity c_P (B10); Q_C is proportional to T_C with the factors mass m_C and specific heat capacity c_C (B11). By equating the first and the second step, the mean body temperature T_M can be described as a function of T_S and T_C (B12): The mean temperature T_M represents the weighted average of the central temperature T_C and the peripheral temperature T_S. The weights are independent of time. Under the further presupposition that the heat capacities of the peripheral and the central component are approximately equal, the weights can be determined as a real number $\gamma \in [0;1]$ for T_C (B13) and $1 - \gamma$ for T_S (B14).

As is evident from the Tables 1 and 2, the estimates for the lower bounds of the production of internal power LPC_I

Table 1 Radiation energy E_R and power P_R, thermal energy E_T and power P_T, lower bound of internal energy LEC_I and power LPC_I in kJ and kJ/h, mean body temperature T_M and skin temperature T_S in °C up to 10 h postmortem (t) for bodies of different size (165 cm / 185 cm) and different stature (50 kg – 65 kg – 80 kg / 70 kg – 85 kg – 100 kg) at outdoor temperature (T_E = 5 °C)

		165 cm; 50 kg							165 cm; 65 Kg							165 cm; 80 kg						
t (h)	T_S	T_M	E_R	E_T	LEC_I	P_R	P_T	LPC_I	T_M	E_R	E_T	LEC_I	P_R	P_T	LPC_I	T_M	E_R	E_T	LEC_I	P_R	P_T	LPC_I
0	33.0	35.8	0	0	0	428	147	363	35.8	0	0	0	478	192	397	35.8	0	0	0	523	236	420
1	30.3	34.7	404	196	290	381	237	227	34.8	451	230	332	426	263	274	34.8	493	265	362	465	291	308
2	27.8	33.1	764	460	469	340	284	138	33.5	853	516	559	380	305	185	33.7	933	575	625	415	326	223
3	25.6	31.4	1085	756	576	303	305	81	32.1	1121	833	711	339	327	123	32.5	1326	911	815	371	345	159
4	23.6	29.6	1371	1064	638	271	309	45	30.6	1533	1165	811	303	335	79	31.2	1676	1261	949	331	354	111
5	21.8	27.9	1628	1370	670	242	302	23	29.1	1819	1500	874	271	333	49	29.9	1989	1616	1041	296	354	76
6	20.2	26.1	1857	1666	686	217	290	10	27.6	2076	1829	911	243	325	28	28.7	2270	1968	1103	265	349	50
7	18.7	24.5	2063	1948	692	195	274	3	26.2	2305	2149	932	217	314	15	27.4	2521	2313	1142	238	340	31
8	17.4	23.0	2247	2214	693	174	257	0	24.8	2511	2456	942	195	300	6	26.2	2746	2648	1166	213	329	18
9	16.2	21.5	2412	2462	693	157	239	0	23.5	2696	2748	946	175	284	2	25.0	2948	2970	1180	191	316	9
10	15.1	20.2	2561	2692	693	141	222	0	22.3	2862	3024	947	157	268	0	23.9	3130	3279	1186	172	302	4

		185 cm; 70 kg							185 cm; 85 kg							185 cm; 100 kg						
t (h)	T_S	T_M	E_R	E_T	LEC_I	P_R	P_T	LPC_I	T_M	E_R	E_T	LEC_I	P_R	P_T	LPC_I	T_M	E_R	E_T	LEC_I	P_R	P_T	LPC_I
0	33.0	35.8	0	0	0	534	206	435	35.8	0	0	0	581	251	460	35.8	0	0	0	623	295	475
1	30.3	34.8	504	242	370	475	272	311	34.9	549	277	401	518	301	346	34.9	588	314	421	555	331	371
2	27.8	33.6	953	536	634	424	312	220	33.8	1038	595	701	462	333	258	33.9	1113	658	749	495	355	287
3	25.6	32.2	1354	860	818	378	333	153	32.6	1474	938	924	412	351	190	32.8	1581	1020	1001	442	368	220
4	23.6	30.8	1712	1198	945	338	342	104	31.4	1864	1294	1087	368	359	138	31.8	1999	1392	1194	395	374	167
5	21.8	29.4	2032	1540	1030	303	341	69	30.2	2213	1655	1205	330	360	99	30.7	2372	1767	1339	353	375	125
6	20.2	28.0	2318	1878	1086	271	335	44	28.9	2525	2013	1288	295	356	69	29.6	2707	2140	1447	316	371	92
7	18,7	26.7	2575	2208	1121	243	324	27	27.7	2804	2365	1344	264	348	46	28.5	3006	2508	1526	284	364	66
8	17.4	25.3	2805	2526	1142	218	311	15	26.6	3055	2707	1382	237	337	30	27.5	3275	2867	1582	254	355	46
9	16.2	24.1	3012	2829	1152	195	296	7	25.3	3279	3038	1405	213	324	18	26.5	3516	3217	1620	228	344	31
10	15.1	22.9	3197	3117	1157	176	281	3	24.4	3481	3355	1419	191	311	8	25.5	3732	3554	1645	205	332	20

Table 2 Radiation energy E_R and power P_R, thermal energy E_T and power P_T, lower bound of internal energy LEC_I and power LPC_I in kJ and kJ/h, mean body temperature T_M and skin temperature T_S in °C up to 10 h postmortem (t) for bodies of different size (165 cm / 185 cm) and different stature (50 kg – 65 kg – 80 kg / 70 kg – 85 kg – 100 kg) at room temperature (T_E = 20 °C)

		165 cm; 50 kg							165 cm; 65 kg							165 cm; 80 kg						
t (h)	T_S	T_M	E_R	E_T	LEC_I	P_R	P_T	LPC_I	T_M	E_R	E_T	LEC_I	P_R	P_T	LPC_I	T_M	E_R	E_T	LEC_I	P_R	P_T	LPC_I
0	33.0	35.8	0	0	0	214	68	186	35.8	0	0	0	239	89	205	35.8	0	0	0	261	109	218
1	31.7	35.2	203	95	149	192	117	116	35.3	227	110	172	215	128	141	35.3	248	126	188	235	140	161
2	30.6	34.5	385	226	240	172	143	70	34.7	430	251	289	193	152	96	34.8	470	277	326	211	160	117
3	29.6	33.6	584	376	295	155	155	41	34.0	613	410	368	173	164	64	34.2	670	443	426	189	172	84
4	28.7	32.7	695	533	326	139	157	23	33.2	777	577	420	156	169	41	33.6	849	618	497	170	177	59
5	27.8	31.8	827	689	342	125	155	11	32.5	925	746	453	140	169	26	32.9	1011	797	547	153	178	41
6	27.1	30.9	946	841	350	113	149	5	31.7	1058	915	473	126	166	15	32.3	1157	975	580	138	177	27
7	26.4	30.1	1053	986	353	101	141	1	31.0	1177	1077	485	113	160	8	31.6	1287	1150	602	124	173	17
8	25.8	29.3	1150	1122	354	91	132	0	20.3	1285	1234	490	102	153	4	31.0	1405	1320	616	112	168	10
9	25.2	28.6	1236	1250	354	82	123	0	29.6	1382	1384	4893	92	146	1	30.4	1511	1485	624	101	161	6
10	24.7	27.9	1314	1368	354	74	114	0	29.0	1469	1526	493	83	138	0	29.8	1607	1643	628	91	155	3

		185 cm; 70 kg							185 cm; 85 kg							185 cm; 100 kg						
t (h)	T_S	T_M	E_R	E_T	LEC_I	P_R	P_T	LPC_I	T_M	E_R	E_T	LEC_I	P_R	P_T	LPC_I	T_M	E_R	E_T	LEC_I	P_R	P_T	LPC_I
0	33.0	35.8	0	0	0	267	96	225	35.8	0	0	0	291	116	239	35.8	0	0	0	312	137	248
1	31.7	35.3	253	115	191	240	132	161	35.4	276	131	209	261	145	181	35.4	295	148	221	280	158	195
2	30.6	34.7	480	260	327	215	154	114	34.8	523	286	366	234	163	136	34.9	561	314	394	251	172	153
3	29.6	34.1	684	421	423	193	167	86	34.2	745	455	484	211	174	101	34.4	799	491	529	226	181	118
4	28.7	33.4	868	591	490	174	172	55	33.6	945	632	571	189	180	74	33.8	1013	674	633	203	186	91
5	27.8	32.6	1033	764	535	156	173	37	33.0	1125	813	634	170	181	54	33.3	1206	861	712	182	187	69
6	27.1	31.9	1181	936	565	141	170	24	32.4	1286	994	679	153	180	38	32.8	1379	1048	772	164	186	51
7	26.4	31.2	1315	1104	584	127	165	15	31.8	1432	1172	711	138	176	26	32.2	1535	1233	816	148	184	38
8	25.8	30.6	1435	1266	595	114	159	8	31.2	1563	1346	732	124	171	17	31.7	1676	1415	848	133	180	27
9	25.2	29.9	1543	1421	602	103	152	4	30.6	1681	1515	746	112	166	11	31.2	1802	1592	871	120	175	19
10	24.7	29.3	1641	1569	605	93	144	2	30.1	1787	1677	754	101	159	6	30.7	1916	1764	886	108	169	12

and energy LEC_I are considerably higher at an environmental temperature of 5 °C than at 20 °C. This dependence of the lower bounds of the internal power and energy does by no means indicate an analogous dependence of the estimated quantities (internal power and energy). Since the lower bounds are derived by calculations of the radiation energy transfer, the strong dependence of the Stefan-Boltzmann law on the environmental temperature is transmitted to the estimated lower bounds. Since chemical reactions are commonly accelerated at higher temperatures, the rate of internal energy production will increase at higher environmental temperatures. The amount of this increase cannot be estimated from the presented model. The true value for internal energy production rate is therefore most probably lower at an environmental temperature of 5 °C than at 20 °C. The lower bounds estimated only from radiation losses, at an environmental temperature of 5 °C (as in Table 1) are therefore valid for higher environmental temperatures as well.

The value of the improved lower bound for the internal energy amounts to about 1000 kJ for a standard sized individual of 75 kg and is roughly in accordance with results of Lundquist [6]. Lundquist [6] estimated an energy production of 140 kcal (which is 587 kJ) for a standard sized individual of 70 kg from the content of glycogen in the body, assumed as 350 g, with an energy output of 0.4 kcal/g glycogen. According to Lundquist [6], other processes, e.g. the hydrolysis of various phosphorous compounds, will amount to approximately half of the energy, produced by the breakdown of glycogen, leading to a total energy output of about 880 kJ during the early postmortem period (assumed by Lundquist [6] as 10 h). The estimations for non-standard sized bodies (Table 1) show that the amounts of internal energy production increase with increasing body weight and size. According to the estimations of Lundquist [6] this could be explained by an increased content of glycogen.

Altogether the estimations presented underline the significance of radiation as a mechanism of energy transfer from the dead body to the cooler environment and give a conservative estimation of the amounts of energy production due to supravital activity depending on the time since death. The estimations are intended as a basis for experimental measurements of the heat production within the dead body in the early postmortem period.

Appendix A

Environmental temperature assumed to be constant:

$$T_E(t) = T_E = \text{const.} \quad \forall\, t \geq 0 \tag{A1}$$

Definition of thermal energy content Q_T (t):

$$Q_T(t) = m\, c\, T_M(t) \tag{A2}$$

Mean body temperature as weighted average of skin and core temperature:

$$T_M(t) \approx (1 - \gamma)\, T_S(t) + \gamma\, T_C(t) \tag{A3}$$

Power due to radiation according to law of Stefan and Boltzmann:

$$P_R(t) = \varepsilon\, \sigma\, A_R\, (T_S{}^4(t) - T_E{}^4) \tag{A4}$$

Power due to conduction/convection as monotonously falling positive function with time:

$$P_C(t) \geq 0 \text{ and } P_C(t) \downarrow \tag{A5}$$

Energies $E_I(t)$, $E_R(t)$ and $E_C(t)$ (t′ indicating time variable in integral expression)

$$E_I(t) = \int_0^t P_I(t')\, dt' \quad E_R(t) = \int_0^t P_R(t')\, dt' \quad E_C(t) = \int_0^t P_C(t')\, dt' \tag{A6}$$

Definition of *apparent* thermal energy change:

$$E_T(t):= Q_T(0) - Q_T(t) = m\, c\, (T_M(0) - T_M(t)) \tag{A7}$$

Balance equation for the energies:

$$E_T(t) + E_I(t) = E_R(t) + E_C(t) \tag{A8}$$

Balance equation for the powers:

$$P_T(t) + P_I(t) = P_R(t) + P_C(t) \tag{A9}$$

Single exponential model for skin cooling:

$$\frac{T_S(t) - T_E}{T_S(0) - T_E} = e^{-Z' t} \tag{A10}$$

Determination of mean body temperature:

$$T_M(t) = 0.3\, T_S(t) + 0.7\, T_C(t) \tag{A11}$$

Double-exponential approach of Marshall and Hoare [9] and Henßge [3] for rectal cooling:

$$T\,(t) = \frac{1}{p - Z}\,(pe^{-Zt} - Ze^{-pt})(T(0) - T_E) + T_E \tag{A12}$$

Solving (A9) for the internal power P_I (t) leads to:

$$P_I(t) = P_R(t) - P_T(t) + P_C(t) \qquad \forall\, t \geq 0 \tag{A13}$$

The following inequality can directly be derived from (A13), since $P_C(t)$ is positive:

$$P_I(t) \geq P_R(t) - P_T(t) \qquad \forall\, t \geq 0 \tag{A14}$$

It can be transferred to the corresponding energies:

$$E_I(t) \geq E_R(t) - E_T(t) \qquad \forall\, t \geq 0 \tag{A15}$$

Since the internal energy E_I accumulates with time, the following estimation is valid for the lower bound of the internal energy (t′ indicating time variable in integral expression):

$$LE_I(t):= \max \left\{ \int_0^{t^*} P_R\,(t') - P_T(t')\, dt' \;\middle|\; t^* < t \right\} \tag{A16a}$$

Substituting (A10) and (A7) provides a calculable formula:

$$LE_I(t):= \max \left\{ \int_0^{t^*} \varepsilon\, \sigma\, A_R\, (T_S(t')^4 - T_E{}^4)\, dt' - (m\, c\, (T_M(0) - T_M(t^*)) \;\middle|\; t^* t \right\} \tag{A16b}$$

In analogy, the lower bound for the internal power is:

$$LP_I(t):= \max \{P_R(t) - P_T(t), 0\} \tag{A17a}$$

after inserting (A10) and (A7):

$$LP_I(t):= \max \left\{\varepsilon\, \sigma\, A_R\, (T_S(t)^4 - T_E{}^4) - \frac{d}{dt}(m\, c\, (T_M(0) - T_M(t))), 0\right\} \tag{A17b}$$

It is possible to determine a time t_{max} with the following quality:

$$(P_T - P_R)(t_{max}) \geq (P_T - P_R)(t) \qquad \forall\, t \geq 0 \tag{A18}$$

Solving the balance equation for powers (A9) at time t_{max} for P_C (t) leads to:

$$P_C(t_{max}) = P_T(t_{max}) - P_R(t_{max}) + P_I(t_{max}) \tag{A19}$$

Since the internal power P_I cannot assume negative values at time t_{max}, the following inequality is valid:

$$P_C(t_{max}) \geq P_T(t_{max}) - P_R(t_{max}) \tag{A20}$$

The power due to conduction/convection represents a monotonously falling function of time:

$$t < t' \Rightarrow P_C(t) < P_C(t') \quad (A21)$$

Therefore, it can be deduced from (A20) for times $t \leq t_{max}$:

$$P_C(t) \geq P_C(t_{max}) \geq P_T(t_{max}) - P_R(t_{max}) \qquad \forall\, t \leq t_{max} \quad (A22)$$

Going back to the balance equation for powers the following inequalities can be formulated:

$$\begin{aligned} P_I(t) &= P_C(t) + P_R(t) - P_T(t) \\ &\geq P_C(t_{max}) + P_R(t) - P_T(t) \\ &\geq (P_T(t_{max}) - P_R(t_{max})) + (P_R(t) - P_T(t)) \qquad \forall\, t \leq t_{max} \quad (A23) \end{aligned}$$

The improved lower bound for the internal power LPC_I then is:

$$P_I(t) \geq LPC_I(t) := (P_T(t_{max}) - P_R(t_{max})) + LP_I(t) \qquad \forall\, t \leq t_{max} \quad (A24)$$

Since the internal energy E_I is the time integral of the internal power P_I, the improved lower bound LEC_I for the internal energy E_I is:

$$E_I(t) \geq LEC_I(t) := (P_T(t_{max}) - P_R(t_{max}))\, t + LE_I(t) \qquad \forall\, t \leq t_{max} \quad (A25)$$

Appendix B

For the very early postmortem period the time dependent functions $E_T(t)$ and $E_R(t)$ can be approximated by a Taylor series expansion of order 1 since the functions E_T and E_R are differentiable functions of time:

$$E_T(t) = \frac{dE_T}{dt}(0)\, t,\ E_R(t) = \frac{dE_R}{dt}(0)\, t \qquad \text{for small } t \quad (B1)$$

For small time spans postmortem the lower bound LE_I (t) becomes equal to zero if E_T (t) = E_R (t) since the functions of $E_T(t)$ and $E_R(t)$ can be approximately assumed to be linear. The limiting condition $LE_I(t) = 0$ can therefore be substituted by the following equation:

$$\frac{dE_T}{dt}(0) = \frac{dE_R}{dt}(0) \quad (B2)$$

By means of a Taylor series expansion in t of order 1 for the functions E_R (t), E_T (t) and T_M (t) , γ_{lim} can be expressed as:

$$\gamma_{lim} = 1 - \frac{\varepsilon\sigma A_R}{Z'mc} \frac{T_S(0)^4 - T_E{}^4}{T_S(0)^4 - T_E} \quad (B3)$$

This formula is obtained in four steps:

Firstly, insert (A12) and (A10) in (A3). A Taylor series expansion of order 1 at time t = 0 leads to:

$$T_M(t) = (\gamma\, T_C(0) + (1 - \gamma) T_S(0)) - (1 - \gamma)(T_S(0) - T_E)\, Z'\, t \quad \text{for small time spans t pm} \quad (B4)$$

Secondly, insert (B4) in (A7):

$$E_T(t) = mc(1 - \gamma)(T_S(0) - T_E)\, Z'\, t \quad \text{for small time spans t pm} \quad (B5)$$

Thirdly, insert (A4) in (A6). A Taylor series expansion of order 1 at time t = 0 leads to:

$$E_R(t) = \varepsilon\, \sigma\, A_R\, (T_S(t)^4 - T_E{}^4)\, t \quad \text{for small time spans t pm} \quad (B6)$$

Fourthly, equalize (B6) and (B5) according to (B2):

$$mc(1 - \gamma)(T_S(0) - T_E)\, Z' = \varepsilon\, \sigma\, A_R\, (T_S(t)^4 - T_E{}^4) \quad (B7)$$

(B7) can be solved for γ. Changing the symbol γ to γ_{lim} (the formula is valid for the limit case $LE_I(t) = 0$) produces the desired formula (B3).

The thermal energy content Q of the body is:

$$Q = m\, c\, T_M \quad (B8)$$

Q can also be expressed as sum of the energy content Q_P of the peripheral and Q_C of the central component:

$$Q = Q_P + Q_C \quad (B9)$$

With the mass of the peripheral component m_P, the mass of the central component m_C and the specific heat capacities c_P for the periphery and c_C for the centre, the energy contents are:

$$Q_P = m_P\, c_P\, T_S \quad (B10)$$
$$Q_C = m_C\, c_C\, T_C \quad (B11)$$

Inserting (B8), (B10) and (B11) in (B9) leads to:

$$T_M = \frac{m_P\, c_P}{(mc)} T_S + \frac{m_C\, c_C}{(mc)} T_C \quad (B12)$$

Compare with (A3), under the assumption that $c \approx c_P \approx c_C$:

$$\gamma = \frac{m_C\, c_C}{(mc)} = m_P/m = (m - m_C)/m = 1 - (m_C/m) \quad (B13)$$

$$1 - \gamma = \frac{m_P\, c_P}{(mc)} = m_C/m \quad (B14)$$

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